

Dynamic Peer-to-Peer Competition

L. F. Caram^a, C. F. Caiafa^{a,b}, A. N. Proto^{a,c} and M. Ausloos^d

^a*Lab. de Sistemas Complejos, FI-UBA. Av. Paseo Colón 850, C1063ACV, Buenos Aires, Argentina.*

^b*LABSP, RIKEN Brain Science Institute, Wako, Saitama 351-0198, Japan.*

^c*Comisión de Investigaciones Científicas PBA(CIC). Av. Paseo Colón 751, C1063ACV, Buenos Aires, Argentina.*

^d*GRAPES@SUPRATECS. B5a, ULG, Sart Tilman, B-4000, Liège, Belgium.*

Abstract

The dynamic behavior of a multiagent system in which the agent size s_i is variable it is studied along a Lotka-Volterra approach. The agent size has hereby for meaning the fraction of a given market that an agent is able to capture (market share). A Lotka-Volterra system of equations for prey-predator problems is considered, the competition factor being related to the difference in size between the agents in a one-on-one competition. This mechanism introduces a natural self-organized dynamic competition among agents. In the competition factor, a parameter σ is introduced for scaling the intensity of agent size similarity, which varies in each iteration cycle. The fixed points of this system are analytically found and their stability analyzed for small systems (with $n = 5$ agents). We have found that different scenarios are possible, from chaotic to non-chaotic motion with cluster formation as function of the σ parameter and depending on the initial conditions imposed to the system. The present contribution aim is to show how a realistic though minimalist nonlinear dynamics model can be used to describe market competition (companies, brokers, decision makers) among other opinion maker communities.

1 Introduction

Multiagent systems and complex networks are very active and growing areas of research [1,2,3] with applications found in a diverse variety of problems found in e.g. engineering systems [4], biological systems [5,6,7,8], neural networks [9], socioeconomic systems [10,11,12,13,14,15].

Email address: `fcaram@fi.uba.ar` (L. F. Caram^a, C. F. Caiafa^{a,b}, A. N. Proto^{a,c} and M. Ausloos^d).

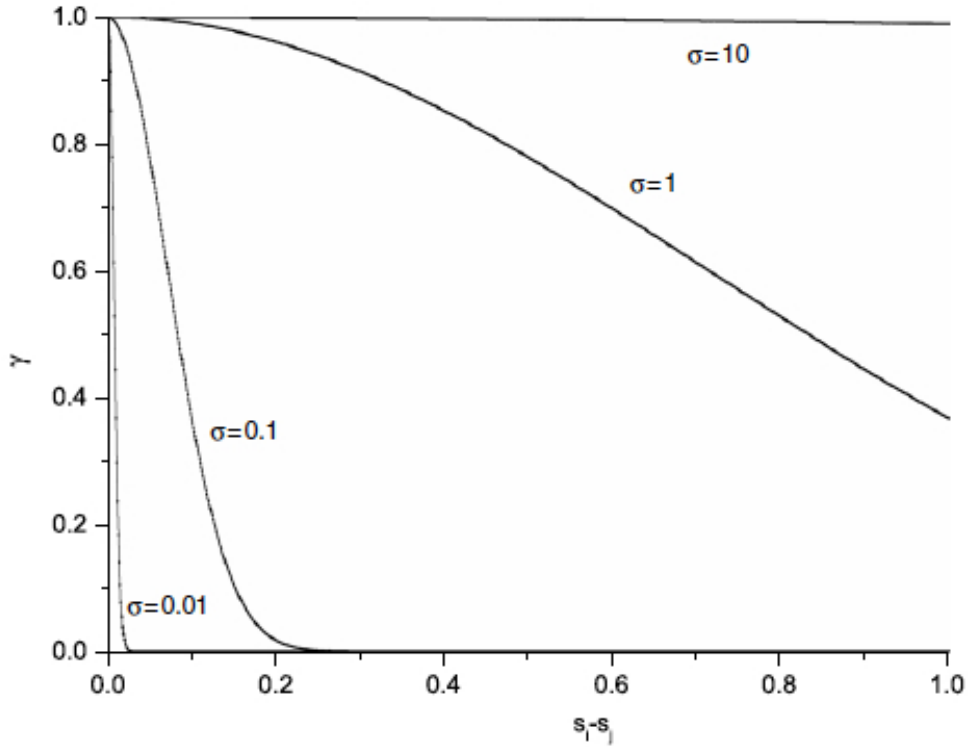


Fig. 1. Interaction function γ vs. agent difference in size ($s_i - s_j$), for different scaling similarity parameter (σ) values.

Let it be recalled that growth, innovation pace of life in cities are scaled using the population size [16] as urban metrics. Similarly, biological metrics such as number, size and metabolic rate of cells are also scaled using body sizes [17,18,19]. In multimarket economies, size is also relevant beside the number of competitors, entry barriers, etc. [20]. Thus many phenomena, outside well known condensed matter physics, related to size, appear in areas that, at first sight, do not have something in common; like sociology, biology, economy, etc., but for which similar mathematical models can be used in fact [21,22].

In complex networks, individuals can behave in two opposite ways: as cooperators or defectors as stated in the N-person prisoner dilemma [23]. Recently much attention has been given to the emergence of "cooperation" in public goods games [24,25], and more generally in "evolution theory". As has been used for conflict resolution according to game theory [26], as well as reference [27,28] in the so-called theory of the organization. There, they treat the conflict between groups within an organization and conflict between organizations, these conflicts, which could be interpreted as a situation of competition for example, are produced by: conflicting objectives and have different perceptions of reality. So the conflict occurs when an agent or agent group experiences a decision problem. Negotiation and politics are the procedures used by the organization to manage these conflicts and lead to tensions in status and power

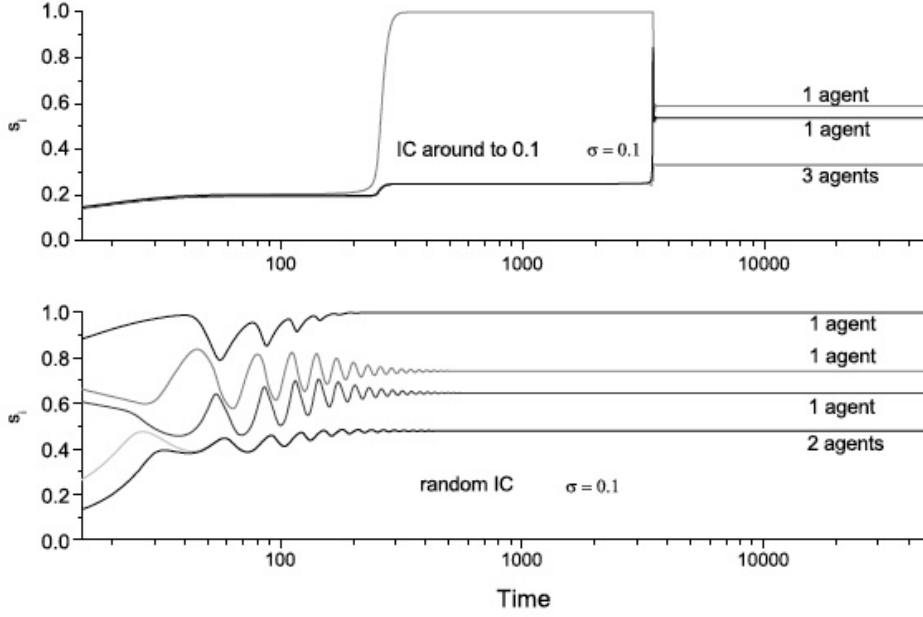


Fig. 2. Two possible simulations of the time evolution of the size of agents for two different sets of initial conditions (IC), both for $\sigma = 0.1$, are shown. On top, it is shown when agent sizes are initially close to each other (Table 1, case IV). On bottom, it is shown for widely spread IC (Table 1, case VI).

systems. If those who are formally more powerful prevail, they give rise to a stronger perception of differences in status and power, if not sustained its position is weakened. To address the conflict between organizations has used the theory of negotiation, albeit with a gap that empirical and negotiation situations are so complex that it is not possible to develop a theory general. Within this framework we can also quote [29,30]. Which defines five forces that determine the competitive strategy of an organization, within them the bargaining power and market threats result in the rivalry of existing competitors. Finally one of the latest Nobel laureate in economics, [31,32,33] referred to the formation of firms, companies, rather than individual goods and services marketed by itself, this is given by the structure of transaction costs.

In the present paper, we focus on a competitive scenario based on a set of differential equations as it will be explained in Section 2. In short, we start from a Lotka-Volterra set of equations for describing prey-predator [34] situations [6,35], i.e. a system of n agents competing for some common resource. An original constraint is imposed on the agent dynamics, i.e. an increase in size is favored in order to obtain resources, while an agent decreasing size implies the loss or lack of getting such resources. By size we mean something like the fraction of a given market that an agent has, i.e. its market share in economy. Other intuitive notions of size are easily imagined in many other systems.

Let us mention that in [11], a different generalization of the Lotka-Volterra

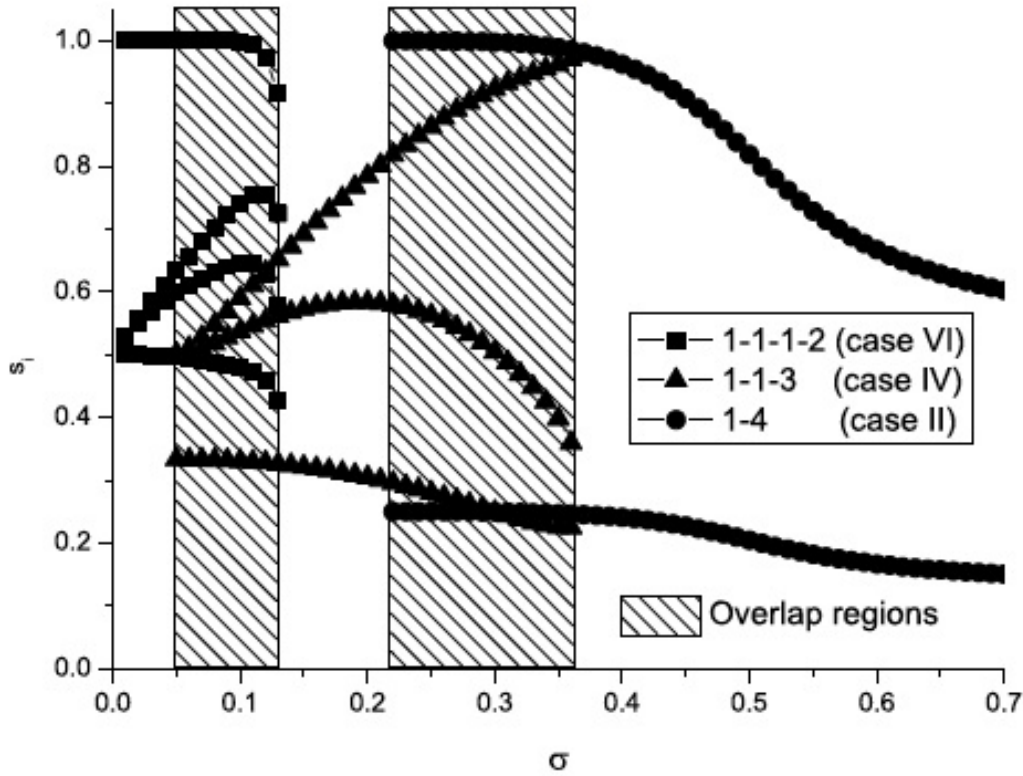


Fig. 3. Agent sizes s_i vs. σ values, in the $n = 5$ case. In the overlap region $0.05 \leq \sigma \leq 0.13$, cases VI and IV coexist. Similarly in the overlap region $0.22 \leq \sigma \leq 0.36$, cases IV and II coexist; see notations in Table 1.

model [35] to the n agent (competitor) case was used for describing the information electronic web's dynamics. Two behavior states were found depending on a global interaction parameter: on one hand, a stationary state exists in which only one agent gets all the available resources ("winner takes all") [11,12], while the rest of agents gets nothing; on the other hand, there is another stationary state where the resource is fairly shared among all agents ("sharing the market"). In several works [10,23,36,37,38], spatial configurations of agents were also considered allowing them to interact only with a limited set of neighbors, thereby resulting in spatially distributed patterns as stationary states of the system [10]. Let us also finally cite [39] in which the interaction coefficient of the model of [11] has a linear form, giving rise to the effect "riches get richer".

Practically we introduce an interaction function (IF) in the Lotka-Volterra model which depends on an agent size in order to let the agents compete among themselves in a self-adjusting way. This contrasts with assuming a fixed interaction coefficient like in the previously cited references. That means that the competition process and outcome are now intertwined variables since they are related to changes in the size of agents. A parameter σ that governs

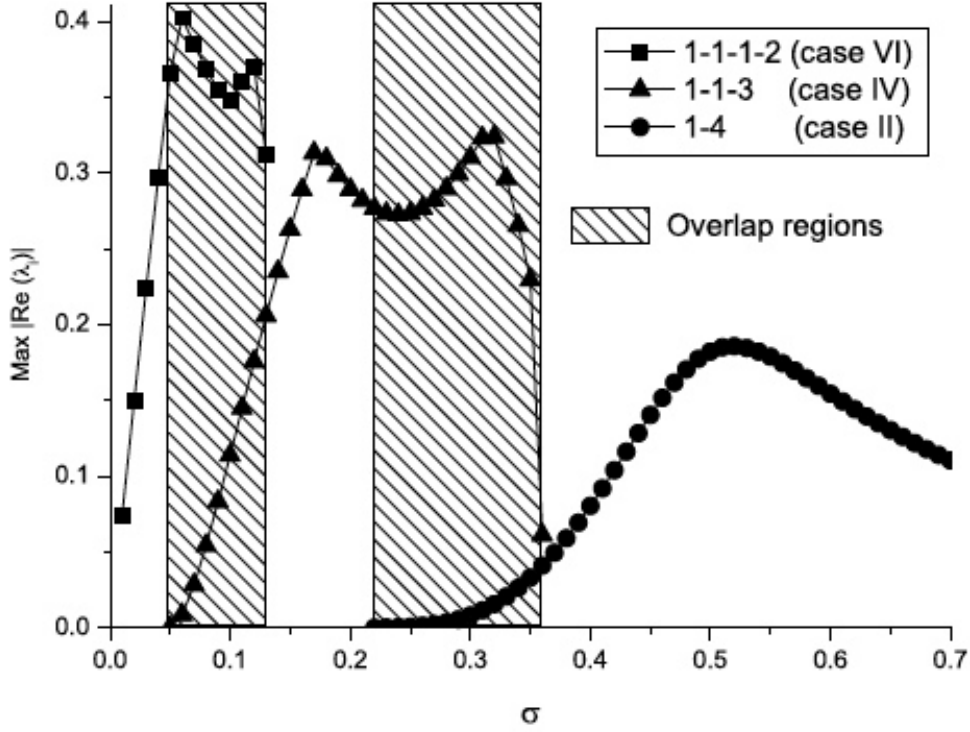


Fig. 4. Maximum absolute of eigenvalue real part of the Jacobian matrix vs. σ value when $n = 5$. The overlap regions $0.05 \leq \sigma \leq 0.13$ of eigenvalues for cases VI and IV, and $0.22 \leq \sigma \leq 0.36$, corresponding to cases IV and II are shown. See notes in Table 1 for following the case properties.

the spread in competition strength, i.e. for scaling the difference between the size of agents, is also introduced. The IF modification will be shown to lead to a dynamics of the system characterized by a time dependent competition. As σ is kept fix along each simulation, we are able to see how different agent groups compete among themselves. In short, this means that the IF is varying with time, accordingly with the size of agents is changing, implies an intrinsically novel dynamics of competition.

This paper is organized as follows: in Section 2, the mathematical model is outlined; in Section 3, the system fixed points are theoretically analyzed and an illustrative example for a system with a small number (five) of agents is provided; in Section 4, simulation results are presented for the case of a larger number (ten) of agents, in order to establish the qualitative validity of the quantitative findings obtained for the small size systems. This allows us in Section 5, to outline the novelty of this work and emphasize some conclusion.

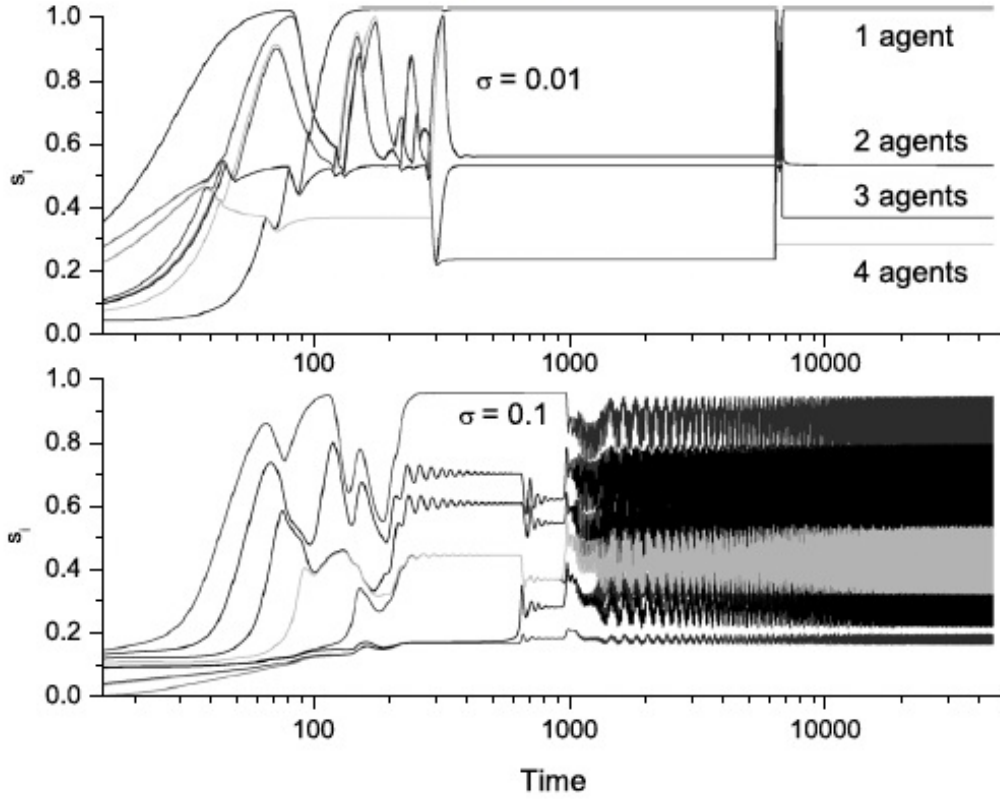


Fig. 5. Simulation of the time evolution of the size of 10 agents for two values of σ . On top, it is shown for $\sigma = 0.01$, there are four clusters each containing 1, 2, 3 and 4 agents respectively. On bottom, it is shown for $\sigma = 0.1$, see the change in asymptotic behavior when σ grows, - from top to bottom.

2 Peer-to-peer competition model

Let us consider a system with n agents competing for some common resource, as in a generalized predator-prey model or Lotka-Volterra model studied in [11]. On this model, a competition parameter, γ , is introduced, which allows the agents to compete among themselves in a self-adjusting way. The system is governed by the following set of n differential equations:

$$\dot{s}_i = \alpha_i s_i (\beta_i - s_i) - \sum_{j \neq i} \gamma(s_i, s_j) s_i s_j \quad \text{for } i = 1, \dots, n \quad (1)$$

where s_i is the size of agent i in the range $0 < s_i \leq 1$; \dot{s}_i is its time derivative; α_i is the growth rate of agent i if no interaction is present; β_i is the maximum capacity of agent i and $\gamma(s_i, s_j)$ is the IF hereby defined by:

$$\gamma(s_i, s_j) = \exp \left[- \left(\frac{s_i - s_j}{\sigma} \right)^2 \right] \quad (2)$$

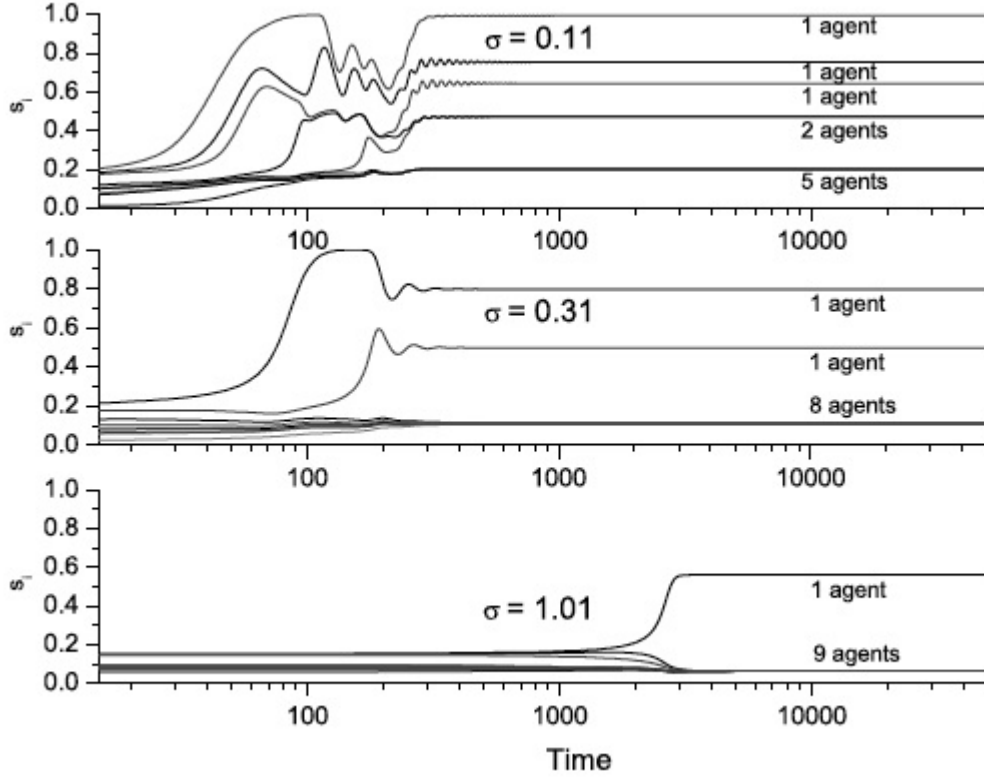


Fig. 6. Simulation of the evolution of the size of ten agents for three different σ values, greater than in Fig.5, i.e. $\sigma = 0.11, 0.31$, and 1.01 . Observe that the amount of agents can vary on various levels, though keeping the total sum of agents equal to 10; the oscillatory asymptotic behavior has disappeared

in which σ is a global positive parameter which controls/scales the s_i degree of similarity in the competition.

Notice that the analytical form in Eq.(2), resembling the Gauss function used in statistics, has been chosen because it allows both analytical and numerical approaches; it has many attractive mathematical properties, i.e., it is a continuously differentiable function allowing us to make a proper theoretical analysis of the system dynamics. In fact, one could simply require any positive and even function of the absolute difference of agent sizes $\Delta = |s_i - s_j|$ with the property that its maximum is located at $\Delta = 0$ and it is a decreasing function for positive Δ . We suggest that another case could be the Kac potential [40]. Other forms can be imagined but to compare them is not the primary purpose of this paper.

From Eq.(2), we see that the interaction is symmetric and always positive representing a competitive and fair scenario; the competition factor is maximum and equal to one when $s_i = s_j$. Additionally, we observe that, as the absolute difference of agent sizes $|s_i - s_j|$ becomes large the competition factor

tends to be small. Plots of the IF $\gamma(s_i, s_j)$ versus the absolute difference of two agent sizes for different values of σ are shown in Fig.1 in order to suggest quantitatively reasonable values.

As we let the interaction coefficient depends on the difference in agent sizes, a peer-to-peer competition is imposed, in the sense that agents with a similar size compete more aggressively than agents with different sizes. The name of peer-to-peer competition model makes sense in this context, as only agents with similar sizes are able to compete reciprocally.

We emphasize that this kind of competition is a phenomenon that can be observed in many socioeconomic systems, i.e. the competition is strong among big companies while the competition is weak between a big company and a medium or small one. Similarly for banks, hedge funds, pension funds, countries, etc. Even in poker game, it is the money the player has at his/her disposal which allows him/her to compete and influence the game.

As it will be shown below, different stationary states emerge as a function of σ , which are interestingly characterized by agent grouping. Self-organized clustering as well as different possible scenarios appear, ranging from chaotic to non-chaotic motion, depending not only on the value of the size similarity measure but also on the agent size initial condition¹.

It should be further noticed that, as σ tends to be larger ($\sigma \rightarrow \infty$) the similarity of competition tends to be static, i.e. $\gamma(s_i, s_j) \rightarrow 1$ constant, approaching to the particular case of "winner takes the maximum, or almost all".

In this paper we consider that all agents have the same dynamics properties restricting our study to the case where $\alpha_i = 1$ and $\beta_i = 1$ for which the equations are:

$$\dot{s}_i = s_i(1 - s_i) - \sum_{i \neq j} \gamma(s_i, s_j) s_i s_j \quad \text{for} \quad i = 1, \dots, n \quad (3)$$

3 Fixed points analysis

First, the existence of fixed points and a study of their stability are presented. By definition, a fixed point is a point in the phase space where all the time derivatives are zero, i.e.,

$$\dot{s}_i = 0 \quad \text{for} \quad i = 1, \dots, n. \quad (4)$$

¹ By initial condition we mean the set of initial agent size values, at simulation time equal to zero.

3.1 Trivial fixed points for n arbitrary agents

From Eq.(4), we detect at least three trivial fixed points which are:

- (I) $s_i = 0$ for $i = 1, \dots, n$ (all agents with zero size);
- (II) $s_i = 1$ and $s_j = 0$ for every $j \neq i$;
- (III) $s_i = b$ for $i = 1, \dots, n$ (all agents with the same size b)

In the latter case (III), we can directly calculate the corresponding constant b by using Eq.(3) as follows:

$$0 = b(1 - b) - (n - 1)b^2 = 1 - bn; \text{ whence we necessarily have } b = \frac{1}{n}.$$

As usual, for the analysis of a system fixed points stability, one needs to look at the eigenvalues of the Jacobian matrix J evaluated at the corresponding fixed points. It is shown that the elements of the Jacobian matrix for the system are:

$$[J]_{(i,k)} = \frac{\partial \dot{s}_i}{\partial s_k} = \begin{cases} 1 - 2s_i - \sum_{i \neq j} s_j \gamma(s_i, s_j) \left[1 - \frac{2}{\sigma^2} s_i (s_i - s_j) \right] & \text{for } k = i \\ -s_i \gamma(s_i, s_k) \left[1 + \frac{2}{\sigma^2} s_k (s_i - s_k) \right] & \text{for } k \neq i. \end{cases} \quad (5)$$

If we evaluate the Jacobian matrix at the fixed points (I), from Eq.(5) we obtain the identity matrix; all its eigenvalues are equal to one ($\lambda_i = 1$) and therefore it is an unstable fixed point.

Evaluating Eq.(5) at the second type of fixed point (II) for the case with $s_1 = 1$ and $s_2 = s_3 = \dots = s_n = 0$, we obtain that the Jacobian matrix is:

$$J = \begin{bmatrix} -1 & -a & -a & \dots & -a \\ 0 & 1-a & 0 & \dots & 0 \\ 0 & 0 & 1-a & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & 1-a \end{bmatrix}$$

with $a = \exp(-\sigma^{-2})$. It can be shown that the eigenvalues of J in this case are:

$$\begin{aligned} \lambda_1 &= -1 \\ \lambda_{2,3,\dots,n} &= 1 - a = 1 - \exp(-\sigma^{-2}) \end{aligned} \quad (6)$$

From Eq.(6) we can deduce that this fixed point is not stable since it has $n - 1$ positive eigenvalues; this is neither dependent on the number of agents nor on the value of the parameter σ .

Next, we analyze the stability of the third type of fixed point (III). By following a procedure similar to the previous one, we obtain the following Jacobian matrix:

$$J = \frac{1}{n} \begin{bmatrix} -1 & -1 & -1 & \dots & -1 \\ -1 & -1 & -1 & \dots & -1 \\ -1 & -1 & -1 & \dots & -1 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ -1 & -1 & -1 & \dots & -1 \end{bmatrix}$$

whose eigenvalues are:

$$\begin{aligned} \lambda_1 &= -1 \\ \lambda_{2,\dots,n} &= 0 \end{aligned} \tag{7}$$

which reveals that it is not a stable fixed point.²

Additionally to the fixed points of type (I), (II) and (III) there are many other points that verify the conditions in Eq.(4). These points are found by seeking the roots of a set of n non-linear equations.

3.2 Trivial fixed points for a small finite number of agents

In order to illustrate the analysis we restrict ourselves to examine the case of $n = 5$ agents. Considering the degeneracy of several solutions, seven possible states appear in which a different combination in the number of agents can be involved. In this sense it can be said that such final states can be called "size levels" or in short "levels". Moreover since several agents can be on the same level, it can be said that "clusters" of agents are obtained. Table 1 enumerates these cases and the type of stability; the latter has been determined by numerically evaluating the Jacobian matrix Eq.(5) at the fixed point through a Newton-Raphson (NR) algorithm [41].

² If one wants to be more rigorous, would need to calculate the second order derivatives since the zero eigenvalues do not wholly determine the stability. Our numerical simulations do not show this fixed point to be a stable one.

Let us examine the seven cases obtained through the numerical solution of Eq.(5). They are summarized in Table 1.

- Case I: only one state final size, level, is found to be occupied by the five agents (5).
- Case II and III two levels appear; with two distributions: called (1-4) and (2-3) i.e. made either of 1 agent and a group of 4 agents or a group of 2 agents and a group of 3 agents, respectively.
- Likewise Cases IV and V are two distributions with three levels (1-1-3) and (1-2-2) respectively.
- Case VI has only one distribution with four levels, (1-1-1-2).
- Finally in Case VII again only one distribution is found, now with five levels, (1-1-1-1-1).

Notice that from Eq.(5) the upper levels are always *less* occupied than the lower one. This fact reflects what seems to happen in real life, as "big players" are less numerous than small ones. Observe also that the number of levels and the fixed point stability depend on σ .

It is also found that the result depends on the initial conditions. Indeed, observe in Fig.2, the time evolution of the s_i set illustrated when $\sigma = 0.1$, $n = 5$ and for two different sets of initial conditions. As it can be seen, for these cases, after 1000 iteration (time) steps different states ("levels") s_i values are asymptotically reached. For arbitrary, random, IC we obtain the solution corresponding to case VI, i.e. (1-1-1-2). However when the size initial conditions are imposed to be taken from a narrow range, the resulting solution corresponds to the case IV, i.e. (1-1-3). Since, as in Fig.2, two different stable solutions can exist for the *same* σ value, but for two different sets of initial conditions (IC), this sort of local degeneracy must be stressed: we call it an overlap region. We emphasize that according to the initial conditions the population of the final states can be markedly different. This observation is reinforced when we analyze the agent size s_i *vs.* σ , here below.

In Fig.3, the corresponding agent sizes are shown at the stable fixed points as a function of the parameter σ . In fixed point searching, we have used the NR method, departing from 100000 random initial conditions looking for the stable fixed points. The numerical evaluation of the Jacobian matrix at different σ values has been done through the NR procedure. Any solution that is not a stable fixed point was discarded. So all the stable fixed point solutions are considered to be found. It can be observed that there are *two* overlap regions in σ space where more than one fixed point exists. In the overlap region $0.05 \leq \sigma \leq 0.13$, cases VI and IV coexist. In the overlap region $0.22 \leq \sigma \leq 0.36$, cases IV and II coexist. In simple words, agents with different asymptotic in time s_i value, or final level, can be simultaneously found, for a given σ , thus for a given IF.

Cases	Size at the stationary state	Type	Stability
Case 1	All agents with the same size	one level 5	Non stable
Case II	One agent with one size and the rest of agents with another size (trivial fixed point)	two levels 1 - 4	Stable depending on σ
Case III	Two agents with one size and the rest of agents with another size	two levels 2-3	Non stable
Case IV	Two agents with one size and three agents with the same size	three levels 1-1-3	Stable depending on σ
Case V	One agent with one size, two agents with another size and two agents with another size	three levels 1-2-2	Non stable
Case VI	Two agents with one size and the rest of agents with different sizes	four levels 1-1-1-2	Stable depending on σ
Case VII	All agents with different sizes	five levels 1-1-1-1-1	Non stable

Table 1

The seven cases emerging from the numerical solution of Eq. (5) by the Newton-Raphson algorithm are summarized. Cases are ordered according with the number of final levels and their characteristic stability are enumerated in the last column. Further explanations are in the text.

Given that absolute eigenvalues determine the strength of the attractor, we can search for the value of the maximum absolute of eigenvalue real part of the Jacobian matrix for each fixed point; this result is shown in Fig.4. A similar feature, i.e. overlap regions, as in Fig.3 is observed as a function of σ , depending on the IC. Thus it is found that the dynamics of the system is dominated by the fixed point of the maximum absolute of eigenvalue real part; the precise dynamics *also* depends on the initial conditions.

Recall that Cases I, III, V and VII are not stable. However Cases II, IV and VI show an interesting type of degenerate stability (two possible solutions coexist within specific ranges of σ) as reported in the previous paragraph. While unique solutions are found in: (i) case VI for $\sigma \leq 0.05$, (ii) case IV for $0.13 \leq \sigma \leq 0.22$ and (iii) case II for $0.36 \leq \sigma$. Many numerical investigations lead us to conclude that

- for random initial conditions, i.e. for a widely spread range of initial sizes,

the solution corresponding to the maximum absolute of eigenvalue real part appears.

- when the initial conditions are taken from a non uniform, i.e. narrow, distribution, the solution corresponding to the other eigenvalue emerges.

4 Simulation results for "large" systems

In this section we present some simulation results for a system with $n = 10$ agents, all of them varying between 0 and 1 in size. We have made simulations where the σ_i parameter has been varied in a logarithmic way, from 0.01 up to 1. In summary, one can observe different dynamics, ranging from the trivial result with almost no competition, when σ is small in relation to the difference of agent sizes, passing through oscillatory regimes, cluster formation regimes, up to the state where only one agent is the biggest and the rest of them remain at a very low level state, when σ is large.

To solve our differential equations system, we have used a typical algorithm for numerical integration [41]. The initial size of each agent in each simulation was randomly chosen from a uniform distribution.

The system evolution for ten agents for $\sigma = 0.01$ and $\sigma = 0.10$ is shown in Fig.5. When σ grows, a change in the system behavior occurs. For $\sigma = 0.01$, four size levels having 1.0, 0.5, 0.33 and 0.25 values are obtained populated by 1, 2, 3 and 4 agents respectively. It is found again that small size agents form more populated clusters. The clusters are formed after a transitory regime. A similar highly complex behavior can be seen, up to around 500 iterations for $\sigma = 0.10$. For higher time iterations some erratic, oscillations appear. No asymptotically strictly stable in time solution exists. For very large time one can consider that a non stable stationary state is reached; it can be conjectured that the corresponding fixed points are non stable either. The evolution goes toward a band clustering rather chaotic situation.

However if σ is further increased above 0.10 (Fig.6), the system asymptotically reaches a stable stationary state, so that a sort of condensation of agents occurs and a cluster type situation appears, with a densely populated level at the lowest size. For $\sigma = 0.11$, it can be observed that there are 5 clusters (1-1-1-2-5); for $\sigma = 0.31$ there are 3 clusters (1-1-8) and finally for $\sigma = 1.01$ only two clusters are present (1-9). It can be seen that agents abandon the oscillatory behavior, seen at intermediate σ values, in order to form different groups or clusters with strong internal competition; this case corresponds e.g. already to the 1-1-1-2-5 set, and is huge in the last case ($\sigma = 1.01$) when the lowest size cluster is made of 9 agents ; recall that the competition factor is maximum and equal to one when $s_i = s_j$ (Fig.2).

It can be also observed that further increasing σ induces a systematic decrease in the number of levels, - the lowest level being always the more crowded. Our numerical simulation has shown that the situation is found for two levels only, i.e. $\sigma = 1.01$, it emerges when σ is around $\simeq 0.5$. This persists for higher σ values. This behavior in which one agent obtains the highest level while the rest of agents reaches the lowest common level illustrates that some sort of monopoly has been configured as the final state of the competition.

5 Conclusions

The main input contribution of this paper consists in the introduction of a self-organized competition scheme related to agent sizes. The agent size acts into our context, through the IF, as a limitation of the number of possibilities for agent interaction or as a constraint. If the difference in sizes is out of the σ scale they are not able to interact anymore. So they are "constrained" in some sense. This constraint allows that only "players" of similar sizes are able to or truly compete with each other. A quite simple example is given by a poker game when, at the end of the game, only those with the same capacity to bet can be continuing the game. Other cases occur in many finite size markets, like pharmaceutical companies through drugstores, fuel distribution through gasoline stations, clothing and specialty goods in supermarket chains.

In the present analysis of the clustering of agents no use is made of techniques like those used physics literature on synchronization [42,43,44,45,46,47], which have been around for a couple of decades, because we do not use chaos synchronization in our contribution. Such extensions should nevertheless be suggested. Our model leads to describe cases when a strong self-organized feedback scheme essentially occurs, i.e. the number of interacting agents being a somewhat secondary aspect. As we are dealing with a nonlinear system, the initial conditions impose severe constraints leading to specific dynamics and a variety of final stationary states. Indeed several behaviors emerge, going from one extreme in which a few agents compete with each other, passing through oscillations with clustering, up to the case of a "winner takes the top" state, and all others drop out. This latter state reflects the fact that the IF tends to a constant value $= 1$, as the size scale parameter σ increases. It can be said in simple words that this corresponds to when "the competition is at its maximum".

We have stressed the case of a small number n of agents which seems the most reasonable practical case [48,49]. The stability analysis for five agents as presented exemplifies the different dynamics and final state possibilities. Here it was demonstrated that there exists a critical σ of the system, $0.05 \leq \sigma_1 \leq 0.13$ and $0.22 \leq \sigma_2 \leq 0.36$, where the solution is no longer unique. It can be

extrapolated the present findings to other finite number n of agents cases.

As shown in several figures, a clustering phenomenon can be also obtained in which the competition is in fact between clusters of agents. In market language, this situation represents the natural segmentation into big, medium, and small players. We have shown that the segmentation can be extreme, even of the binary type. From a socioeconomic point of view, this means that a monopolistic situation is sometimes likely.

We emphasize the relevance of initial conditions, but agree that in general they are hard to define in socioeconomic systems.

Notice also that even when clusters (in size) appear there still exists competition among the agents *inside* the cluster; this competition is stronger at the lowest size levels, which are the most densely populated. This fact well reflects the complexity of markets: not only agents with equal sizes are in competition with those with bigger or smaller sizes, but also they are still in strong competition with each other at their own level. In some sense, the only final state will be, sooner or later, only one agent at the top, because of the attrition of the small ones.

Finally and in addition we would like to re-emphasize that the competition scheme presented here Eq.(2) gives rise to a very complex dynamics, although based on a simple idea, and on having only one quantity σ , as the regulation parameter of the different behaviors. This can be put in line with the observed complexity of evolution of true systems, though is clearly a reductionist view of the world [50].

Acknowledgements: Thanks to a CONICET - CGRI-FNRS bilateral agreement which has allowed interaction between the authors.

References

- [1] R. Albert, A.L. Barabási, Rev. Mod. Phys. 74 (2002) 47-97.
- [2] L.A.N. Amaral, J.M. Ottino, Eur. Phys. J. B 38 (2004) 147-162.
- [3] H.N. Knyazeva, S.P. Kurdyumov, Evolution and Self-organization Laws of Complex Systems, Moscow, Nauka Publisher in Russian 1994 p 236.
- [4] L. Kocarev, G. Vattay, Complex Dynamics in Communication Networks, Springer Verlag 2005.
- [5] S. Sitharama Iyengar, Computer Modeling and Simulations of Complex Biological Systems, 2nd edn. CRC, 1997.

- [6] D. Arrowsmith, C.M. Place, Dynamical Systems: Differential Equations, Maps and Chaotic Behaviour, Chapman and Hall, 1992.
- [7] R. Bagni, R. Berchi, P. Cariello, J. Artif. Soc. Social Simul. 5 (2002) 3. [http :
//jasss.soc.surrey.ac.uk/5/3/5.html](http://jasss.soc.surrey.ac.uk/5/3/5.html)
- [8] F. Liljeros, C.R. Edling, L. A. N. Amaral, Microbes Infect. 5 (2003) 189-196.
- [9] Zhang Yi, K.K. Tan, IEEE Trans. Circuits Syst. 52 (2005) 2482-2489.
- [10] C.F. Caiafa, A.N. Proto, Intern. J. Mod. Phys. C 17 (2006) 385-394.
- [11] S.M. Maurer, B.A. Huberman, J. Econom. Dynam. Control 27 (2003) 2195-2206.
- [12] L.A. Adamic, B.A. Huberman, Quart. J. Electron. Commerce 1 (2000) 5-12.
- [13] J. Miskiewicz, M. Ausloos, in Proc. of the 3rd Nikkei Econophysics Symposium: Practical Fruits of Econophysics, Tokyo, ed H. Takayasu, Springer 2006, p. 312.
- [14] M. Gligor, M. Ausloos, Eur. Phys. J. B 57 (2007) 139-146.
- [15] F.O. Redelico, A.N. Proto, M. Ausloos, Physica A 388 (2009) 3527-3535.
- [16] L.M.A. Bettencourt, J. Lobo, D. Helbing, C. Kuhnert, G.B. West, Proc. Natl. Acad. Sci. 104 (2007) 7301-7306.
- [17] V.M. Savage, A.P. Allen, J.H. Brown, J.F. Gillooly, A.B. Herman, W.H. Woodruff, G.B. West, Proc. Natl. Acad. Sci. 104 (2007) 4718-4723.
- [18] A.L. Moran, J.D. Allen, Biol. Bull. 212 (2007) 143-150.
- [19] Y. Sawada, Physica A 204 (1994) 543-554.
- [20] S.J. Pilloff, Rev. Industr. Organ. 14 (1999) 163-182.
- [21] E.P. Economo, A.J. Kerkhoff, B.J. Enquist, Ecol.Lett. 8 (2005) 353-360.
- [22] N. Olivera, A.N. Proto, F. Redelico, RC332008 7 th International Conference on Social Science Methodology, Campus di Monte Sant'Angelo, Naples, Italy (2008).
- [23] C. Hauert, S. De Monte, J. Hofbauer, K. Sigmund, Science 296 (2002) 1129-1132.
- [24] F.C. Santos, M.D. Santos, J.M. Pacheco, Nature 454 (2008) 213-216.
- [25] N.S. Glance, B.A. Huberman, Sci Am. 270 (1994) 76-81.
- [26] J. von Neumann, O. Morgenstern, Theory of Games and Economics Behavior, Princenton, 1944.
- [27] J.G. March, H.A. Simon, Organizations, Wiley, New York, 1958.
- [28] H.A. Simon, Administrative Behavior, Free Press, New York, 1947.

- [29] M. Porter, Competitive Strategy, Free Press, New York 1980.
- [30] M. Porter, Competitive Advantage, Free Press, New York 1985.
- [31] R.H. Coase, The firm, the market, and the law Chicago: University of Chicago Press, 1988.
- [32] O.E. Williamson, S.G. Winter, R.H. Coase, The Nature of the firm: Origins, evolution, and development New York: Oxford University Press, 1991.
- [33] O.E. Williamson, The Economic Institutions of Capitalism. Free Press, New York, 1985.
- [34] A. Pekalski, Comp. Sci. Eng. 6 (2004) 62-66.
- [35] A.J. Lotka, Elements of Physical Biology, Williams&Wilkins Co., Baltimore, 1925, p.460.
- [36] G. Szabo, C. Hauert, Phys. Rev. Lett. 89 (2002) 118101.
- [37] G. Szabo, C. Hauert, Phys. Rev. E 66 (2002) 062903.
- [38] L.F. Caram, C.F. Caiafa, A.N. Proto, Intern. J. Mod. Phys. C 17 (2006) 435-445.
- [39] Li Yanhui, Zhu Siming, Appl. Math. Modelling 31 (2007) 912-919.
- [40] M. Kac, Phys. Fluids 2 (1959) 8-12.
- [41] W.H. Press, B.P. Flannery, S.A Teukolsky, W.T. Vetterling, Numerical Recipes in FORTRAN: The Art of Scientific Computing, 2nd edn, Cambridge: Cambridge University Press, 1992, p.364.
- [42] G.B. Ermentrout, N. Kopell, SIAM J. Math. Anal. 15 (1984) 215-.
- [43] S.H. Strogatz, R.E. Mirollo, Physica D 31 (1988) 143-.
- [44] Z. Liu, Y.C. Lai, F.C. Hoppensteadt, Phys. Rev. E 63 (2001) 055201.
- [45] Z. Zheng, B. Hu, G. Hu, Phys. Rev. E 62 (2000) 402-.
- [46] G. Hu, Y. Zhang, H.A. Cerdeira, S. Chen, Phys. Rev. Lett. 85 (2000) 3377-.
- [47] Z. Zheng, B. Hu, G. Hu, Phys. Rev. E 62 (2000) 402-.
- [48] S.S. Manna, Phys. Rev. E 68 (2003) 027104.
- [49] M. Bawa, H. Garcia-Molina, A. Gionis, R. Motwani, Technical report, Stanford, 2003, submitted for publication; [http : //dbpubs.stanford.edu/pub/2003 – 24](http://dbpubs.stanford.edu/pub/2003-24)
- [50] H. Knyazeva, J. Gen. Philoso. Sci. 36 (2005) 289-304.